Matrices

$[A] = [a_{ij}] = A$ → An array of numbers.

row →

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

column

order

row × column

$m \times n$

3×5

entry

a_{ij}

ith row, jth column

a_{23}

Equality

$$\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad a=2 \quad b=-3 \quad c=0 \quad d=1$$

Add/subtracting

$$\begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 0 \\ 6 & 7 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 4 \\ 6 & 8 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 0 & 1 \end{bmatrix} = \not\emptyset$$

Scalar multiplication

$$5\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 10 & 15 \end{bmatrix}$$

Multiplication

$$\begin{bmatrix} 2 & 0 \\ 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 7 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2\cdot 3 + 0\cdot 2 & 2\cdot 7 + 0\cdot -1 \\ 1\cdot 3 + -4\cdot 2 & 1\cdot 7 + -4\cdot -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 14 \\ -5 & 11 \end{bmatrix}$$

Problems

① What number is in ⓐ 3rd row, 2nd column? ⓑ 2nd row 3rd column

$$\begin{bmatrix} 1 & 3 & 6 \\ 27 & 0 & 4 \\ 3 & 9 & 2 \end{bmatrix}$$

② $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 9 & 3 \\ 2 & 1 \end{bmatrix}$

③ $\begin{bmatrix} 5 & 8 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 4 & 7 \end{bmatrix}$

④ $\begin{bmatrix} 2 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

⑤ $\begin{bmatrix} 8 & 2 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 \\ 1 & -5 \end{bmatrix}$

⑥ $\begin{bmatrix} 3 & 0 \\ [illegible] & [illegible] \\ [illegible] & [illegible] \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$

⑦ $\begin{bmatrix} 3 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 3 & 0 \\ -2 & 3 \end{bmatrix}$

⑧ $\begin{bmatrix} 2 & 4 \\ 3 & 0 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -1 \end{bmatrix}$

$$\begin{bmatrix} 3 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 3 & 0 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 3\cdot2+1\cdot3+-1\cdot-2 & 3\cdot4+1\cdot0+-1\cdot-3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 15 \end{bmatrix}$$

1 × 3 ; 3 × 2 ; size of answer

$$\begin{pmatrix} 2 & 4 \\ 3 & 0 \\ -2 & 3 \end{pmatrix} \cdot \begin{bmatrix} 3 & 1 & -1 \end{bmatrix} = \text{undef.}$$

3 × 2 ; 1 × 3 ; ×

$$\begin{aligned} 2x+y-z&=5\\ 3x-y+2z&=-1\\ x-y-z&=0 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & -1 & 5\\ 3 & -1 & 2 & -1\\ 1 & -1 & -1 & 0 \end{array}\right]$$

augmented matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0\\ 3 & -1 & 2 & -1\\ 2 & 1 & -1 & 5 \end{array}\right] \quad \text{switch } R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0\\ 0 & 2 & 5 & -1\\ 2 & 1 & -1 & 5 \end{array}\right] \quad -3R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0\\ 0 & 2 & 5 & -1\\ 0 & 3 & 1 & 5 \end{array}\right] \quad -2R_1 + R_3 \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0\\ 0 & 1 & \frac{5}{2} & -\frac{1}{2}\\ 0 & 3 & 1 & 5 \end{array}\right] \quad R_2 \div 2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0\\ 0 & 1 & \frac{5}{2} & -\frac{1}{2}\\ 0 & 0 & -\frac{13}{2} & \frac{13}{2} \end{array}\right] \Rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0\\ 0 & 1 & \frac{5}{2} & -\frac{1}{2}\\ 0 & 0 & 1 & -1 \end{array}\right] \quad \boxed{z=-1}$$

$$x-y-z=0$$
$$y+\tfrac{5}{2}z=-\tfrac{1}{2}$$
$$z=-1$$

$$y+\tfrac{5}{2}(-1)=-\tfrac{1}{2}$$
$$y-2.5=-0.5$$
$$+2.5 \qquad +2.5$$
$$\boxed{y=2}$$

$$x-2-(-1)=0$$
$$x-2+1=0$$
$$\boxed{x=1}$$

$$2x + y - z = 5$$
$$3x - y + 2z = -1$$
$$x - y - z = 0$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & -1 \\ 3 & -1 & 2 \\ 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$$

coefficient matrix — variable matrix — Answer matrix

$$[A] \cdot [X] = [B]$$

$$[X] = [A]^{-1} \cdot [B]$$

$$\frac{a}{a}x = \frac{b}{a}$$

$$x = \frac{b}{a}$$

$$\frac{1}{a} \cdot ax = b \cdot \frac{1}{a}$$

Decoding

A message was encoded using the matrix $\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$ $[A]$

to give 13 -26 21 33 -53 -12

18 -23 -42 5 -20 56

-24 23 77

$[\#\ \#\ \#] \cdot [A]$

$1 \times 3 \quad 3 \times 3$

$[\#\ \#\ \#] \cdot [A]^{-1}$

M O N I

15 14 4

13 5 0 13

1 25 0

A Y

13 5 5 20 0 13 5 0 13

M E E T M E

7.3 #19, 38

7.4 #2-12 (even), 55, 59
(easy)

7.8, p. 545 #31, 33
see ex. 8